MECHANICS.

75. Proposed by B. F. FINKEL, A.M.,M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A particle, P, is held in a bent tube by two forces directed towards two fixed points, H and S. Show that the equation of the tube is $PS.PH=k^2$, if the forces are μ/PS and μ/PH.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

Let $PS=r$, $PH=r'$, $\mu/PS=f$, $\mu/PH=f'$.

By the principle of virtual work we have for equilibrium,

$$fdr+f'dr'=0, \text{ but } f/f'=r'/r \text{ or } f'r'=fr.$$

Dividing $fdr=-f'dr'$ by $fr=f'r'$ we get

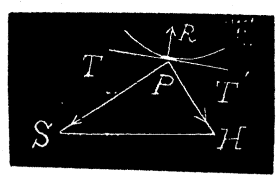
$$dr/r=-dr'/r' \text{ or } rdr'+r'dr=0.$$

Integrating, we get $rr'=$a constant$=k^2$.

$\therefore PS.PH=k^2$.

IV. Solution by GEORGE LILLEY, Ph.D., Professor of Mathematics, University of Oregon, Eugene, Oregon.

Let P be any position of the particle, TT' the tangent to the tube at P, $\angle TPS=\phi$, $\angle TPH=\phi'$, $\angle PSH=\theta$, $\angle PHS=\theta'$, $PS=r$ and $PH=r'$.



Resolve along TT', $\frac{\mu}{r}\cos\phi+\frac{\mu}{r'}\cos\phi'=0$.

But, $\tan\phi=r\frac{d\theta}{dr}$; hence, $\cos\phi=\frac{dr}{ds}$, where ds is element length of the tube. Also $\cos\phi'=\frac{dr}{ds}$.

Therefore, $\frac{dr}{2r}+\frac{dr'}{2r'}=0$.

Integrating, $\log\sqrt{(rr')}+c=0$, where c depends on known values of r and r'.

Therefore, $SP.PH=k^2$.

Or thus : By the method of virtual work,

$Fdr+F'dr'=0$, where $F=\frac{\mu}{SP}$ and $F'=\frac{\mu}{HP}$.

Thus form the differential equation and solve it as above.

V. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Miss.

Let P be the particle, S and H the fixed points. Denote PS, PH, and SH by h, s, and $2m$, respectively. Take the origin of rectangular axes midway between S and H, the x-axis lying along the line SH. Let the resultant force

acting at P (x, y) intersect Ox at A, and make angles α, β, θ, with PS, PH, and AH, respectively.

Since $\frac{\mu}{SP}:\frac{\mu}{HP}=\sin\beta:\sin\alpha$, $HP:SP=\sin\beta:\sin\alpha$,

$$\text{or } \frac{\sin\alpha}{\sin\beta}=\frac{h}{s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1).$$

Also $\frac{s.\sin\beta}{h.\sin\alpha}=\frac{AH}{AS}=\frac{m-OA}{m+OA}$.

Substitute from (1) and solve for OA, obtaining

$$OA=\frac{h^2-s^2}{h^2+s^2}m. \quad \text{Tan}\theta=\frac{y}{x-OA}=\frac{y}{x-\dfrac{h^2-s^2}{h^2+s^2}m}=\frac{y^3+x^2y+m^2y}{x^3+xy^2-m^2x}.$$

Since PA is normal to the tube, the differential equation of the curve is

$$\frac{dy}{dx}=-\frac{x^3+xy^2-m^2x}{y^3+x^2y+m^2y}.$$

Integrating, $y^4+x^4+2x^2y^2+2m^2y^2-2m^2x^2=c$.
Adding m^4 to both members, and factoring,

$$[y^2+(m+x)^2][y^2+(m-x)^2]=c+m^4, \text{ or } SP^2.HP^2=c+m^4=k^4,$$

from which $SP.HP=k^2$.

VI. Solution by R. E. GAINES, Professor of Mathematics, Richmond College, Richmond, Va.

Denote PS and PH by r and r', respectively, and these may be taken as the "bipunctual coördinates" of P. Then it is easy to show that

$$\frac{dr'}{ds}=\cos\psi \text{ and } \frac{dr}{ds}=-\cos\varphi.$$

$$\therefore \frac{dr'}{dr}=-\frac{\cos\psi}{\cos\varphi}.$$

Now resolving forces along the tangent at P we have,

$$\frac{\mu}{r}\cos\varphi=\frac{\mu}{r'}\cos\psi \text{ or } \frac{1}{r}-\frac{1}{r'}\cdot\frac{\cos\psi}{\cos\varphi}=0.$$

$$\therefore \frac{1}{r}+\frac{1}{r'}\frac{dr'}{dr}=0.$$

$$\therefore \log r+\log r'=\log k^2. \quad \therefore rr'=k^2.$$

If the forces had been $\mu f(r)$ and $\mu F(r')$ we could get the form of the curve by integrating

$$f(r)+F(r')\frac{dr'}{dr}=0.$$

76. Proposed by JAMES F. LAWRENCE, Classical Sophomore, Drury College, Springfield, Mo.

An inclined plane of mass M is capable of moving freely on a smooth horizontal plane. A perfectly rough sphere of mass m is placed on its inclined face and rolls down under the action of gravity. If x' be the horizontal space advanced by the incline plane, x the part of the plane rolled over by the sphere, prove that $(M+m)x'=mx\cos\alpha$, $\frac{7}{5}x-x'\cos\alpha=\frac{1}{2}gt^2\sin\alpha$, where α is the inclination of the plane. [From *Routh's Elementary Rigid Dynamics*, page 126.]

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

Let F=friction of the sphere and plane, R=their mutual reaction, θ=the angle through which the sphere has rotated from the beginning of motion, y=the vertical distance of the center of the sphere from the horizontal plane, x_1=the corresponding abscissa, h and k the initial values of x and y, respectively, and a= the radius of the sphere.

For the motion of the sphere, resolving horizontally and vertically, and taking moments about the center of the sphere,

$$m\frac{d^2x_1}{dt^2}=F\cos\alpha-R\sin\alpha\,........(1),\quad m\frac{d^2y}{dt^2}=F\sin\alpha+R\cos\alpha-mg\,.......(2),$$

$$\tfrac{2}{5}ma^2\frac{d^2\theta}{dt^2}=aF\,........(3).$$

For the horizontal motion of the plane,

$$M\frac{d^2x'}{dt^2}=-F\cos\alpha+R\sin\alpha\,........(4).$$

Also, $x_1=h+x'-a\theta\cos\alpha$ ....(5), $y=k-a\theta\sin\alpha$ ....(6).

From (5) and (1), $m\dfrac{d^2x'}{dt^2}-ma\cos\alpha\dfrac{d^2\theta}{dt^2}=F\cos\alpha-R\sin\alpha$ ....(7);

and from (6) and (2), $-ma\sin\alpha\dfrac{d^2\theta}{dt^2}=F\sin\alpha+R\cos\alpha-mg$ ....(8).

Eliminating F and R from (3), (7) and (8),

$$m\cos\alpha\frac{d^2x'}{dt^2}=\tfrac{7}{5}ma\frac{d^2\theta}{dt^2}-mg\sin\alpha\,......(9).$$

Integrating (9), noticing that when $t=0$, $\dfrac{dx^2}{dt}=0$, $\dfrac{d\theta}{dt}=0$, and $x'=0$, $\theta=0$,